Grundlagen der Elektrotechnik

Tief- und Hochpass-Filter

TH-Köln 2021

Prof. Dr. Eberhard Waffenschmidt

Tief- und Hochpass-Filter

- Einfache Frequenzverläufe mit
  - Kondensator
  - Induktivität
- Tiefpassfilter
  - Frequenzverlauf
  - Kennwerte
  - Vergleich Frequenz- und Zeitbereich
- Hochpassfilter
- Übersicht

Einfache Impedanzen

- **Kondensator**
  Spannung fällt linear

$$\underline{U} = \underline{I} \cdot \frac{1}{j\omega \cdot C}$$

$$|\underline{U}| = |\underline{I}| \cdot \frac{1}{2\pi f \cdot C}$$

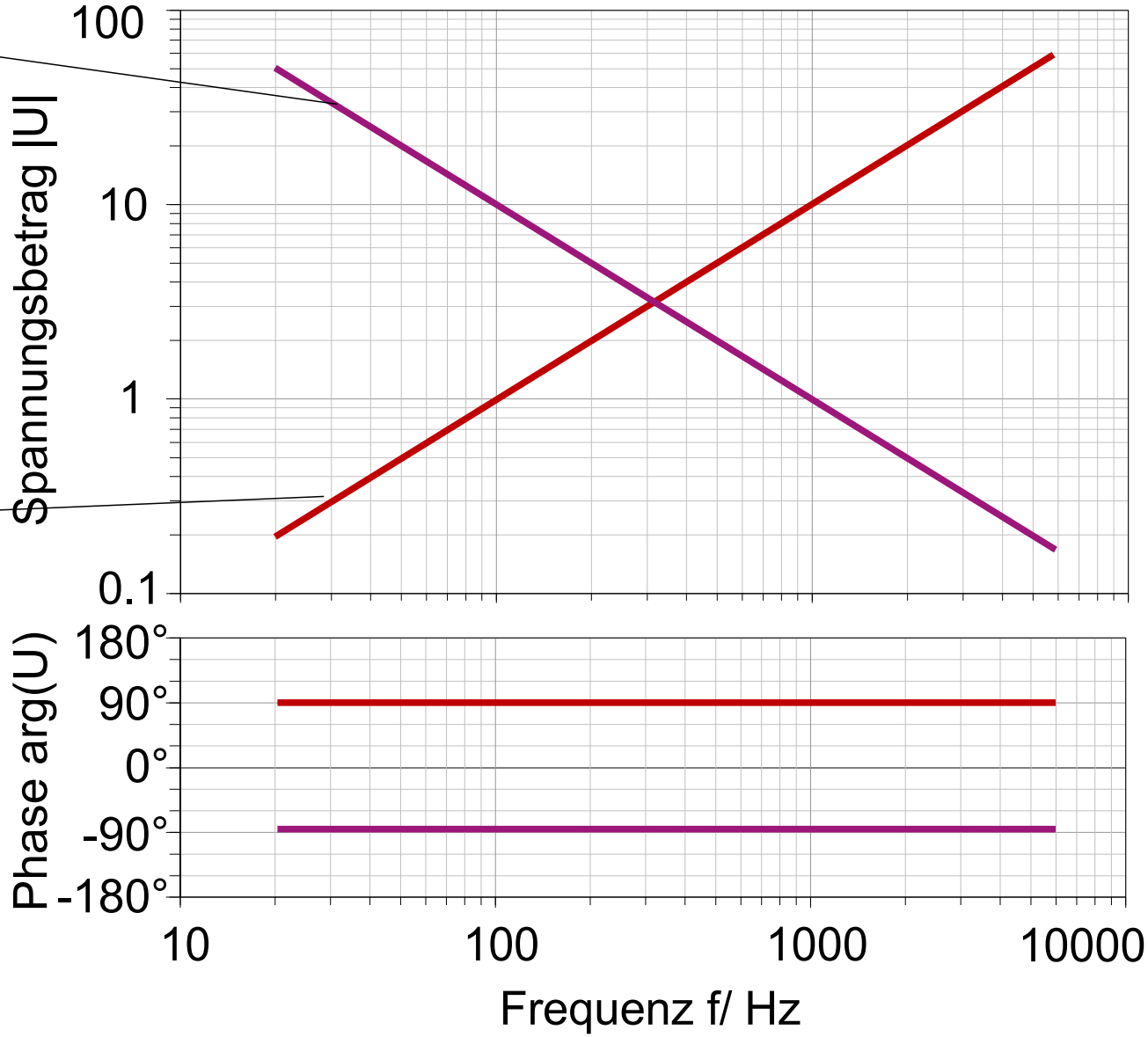
$$arg(\underline{U}) = -90°$$

- **Induktivität**
  Spannung steigt linear

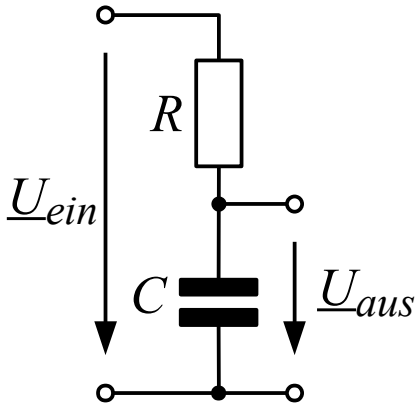
$$\underline{U} = \underline{I} \cdot j\omega \cdot L$$

$$|\underline{U}| = |\underline{I}| \cdot 2\pi \cdot f \cdot L$$

$$arg(\underline{U}) = 90°$$



Tiefpass: Frequenzverlauf

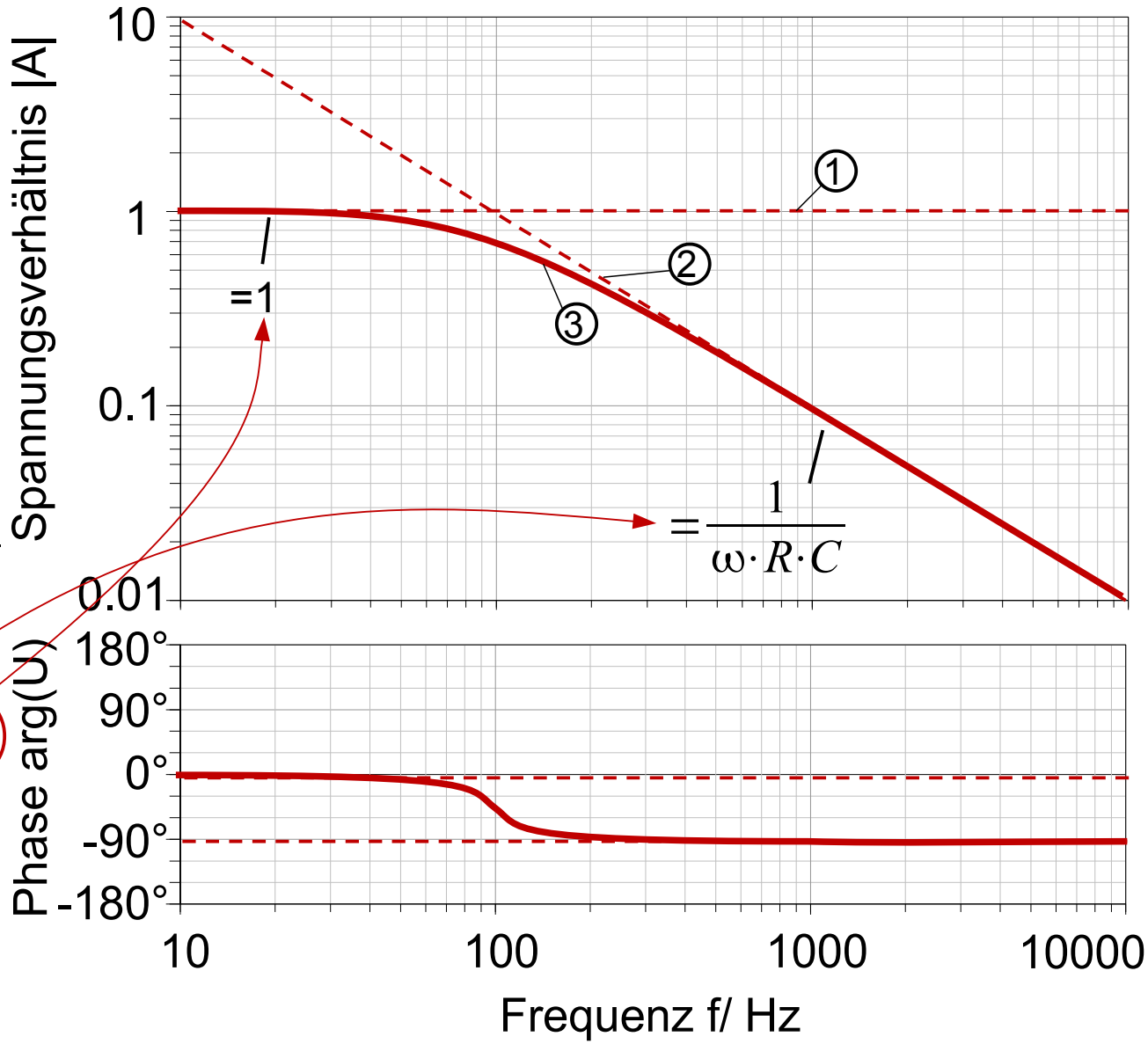


$$\frac{\underline{U}_{aus}}{\underline{U}_{ein}} = \underline{A} = \frac{1/j\omega C}{R+1/j\omega C}$$

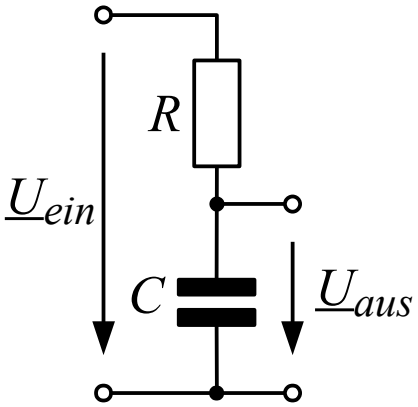
$$= \frac{1}{j\omega \cdot C \cdot R+1} = \frac{1}{\frac{1}{1/j\omega RC}+\frac{1}{1}}$$

$$= 1/j\omega RC \parallel 1$$

$$|\underline{A}| = \frac{1}{\sqrt{\left(\frac{1}{1/\omega RC}\right)^2+\left(\frac{1}{1}\right)^2}}$$



Tiefpass: Charakteristische Werte



Schnittpunkt der Geraden:

Grenzfrequenz: $\omega_0 = 2\pi f_0$

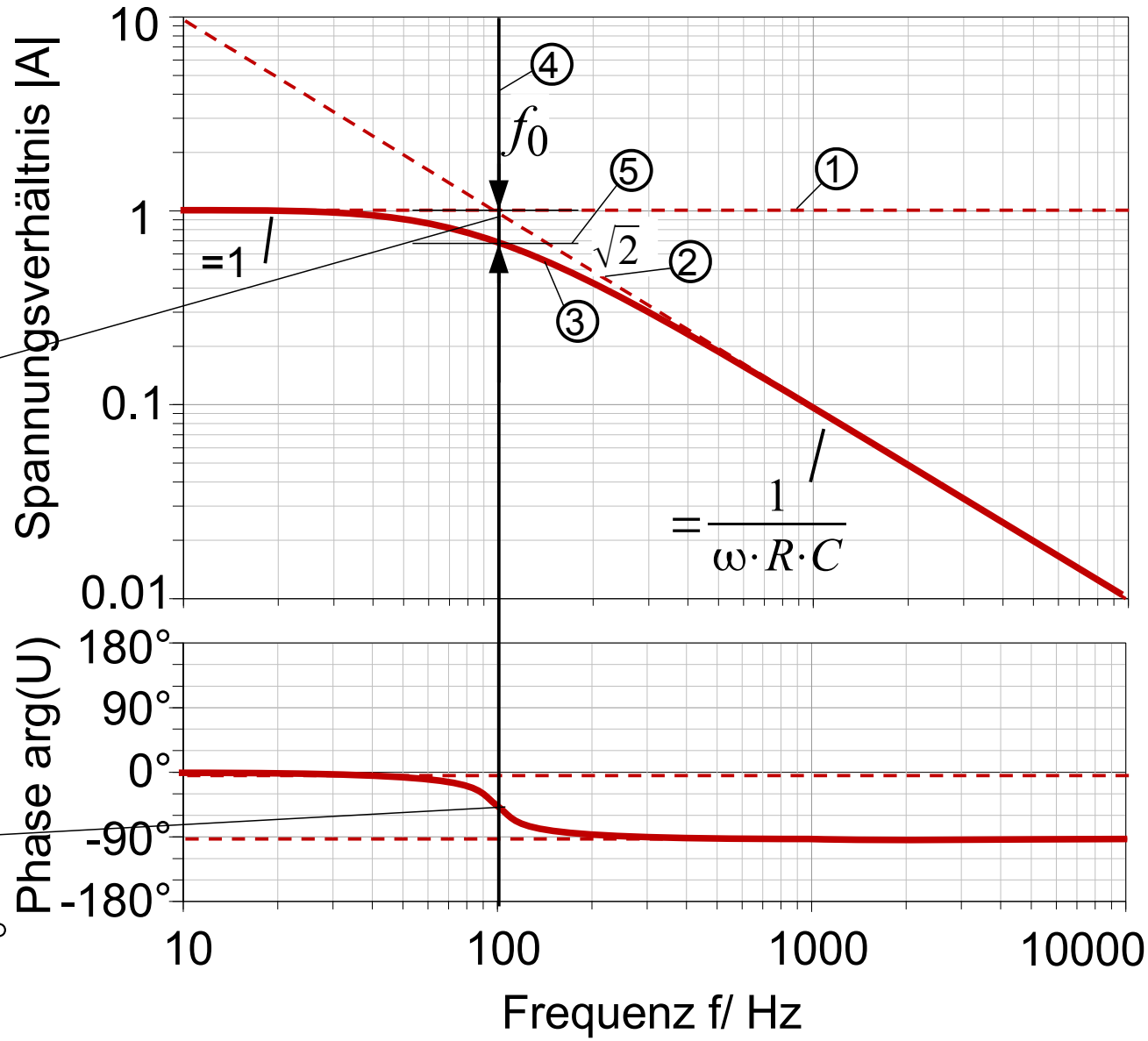
$$1 = \frac{1}{\omega_0 \cdot R \cdot C} \quad \rightarrow \omega_0 = \frac{1}{R \cdot C}$$

Abweichung von Geraden:

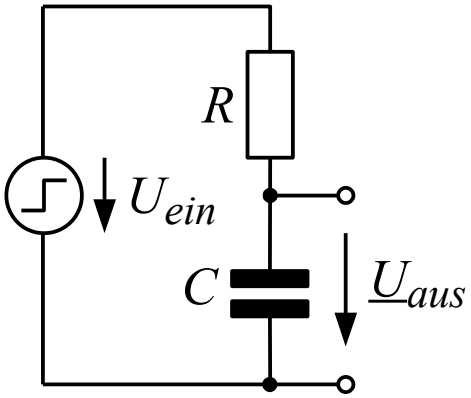
$$|\Delta A| = \sqrt{2}$$

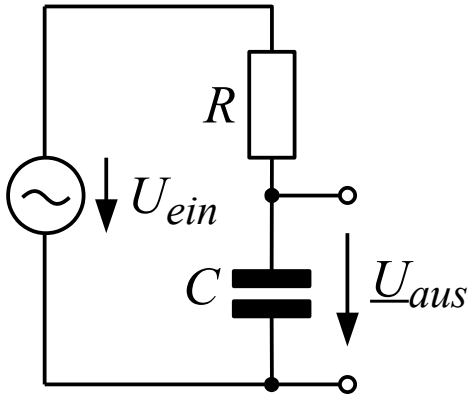
Phase:

$$\mathrm{Re}(A) = \mathrm{Im}(A) \quad \rightarrow arg(A) = -45°$$



Tiefpass: Zeit- und Frequenzbereich





Kennwert:
Zeitkonstante

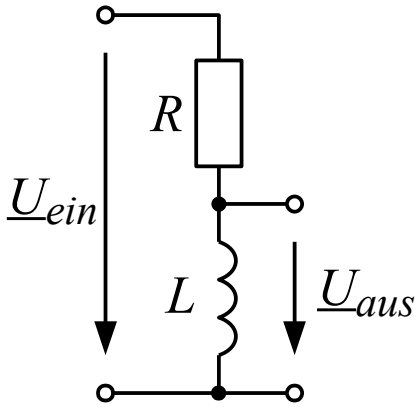
$$\tau = R \cdot C$$

Kennwert:
Grenzfrequenz

$$\omega_0 = \frac{1}{R \cdot C}$$

$$\tau = \frac{1}{\omega_0}$$

Hochpass

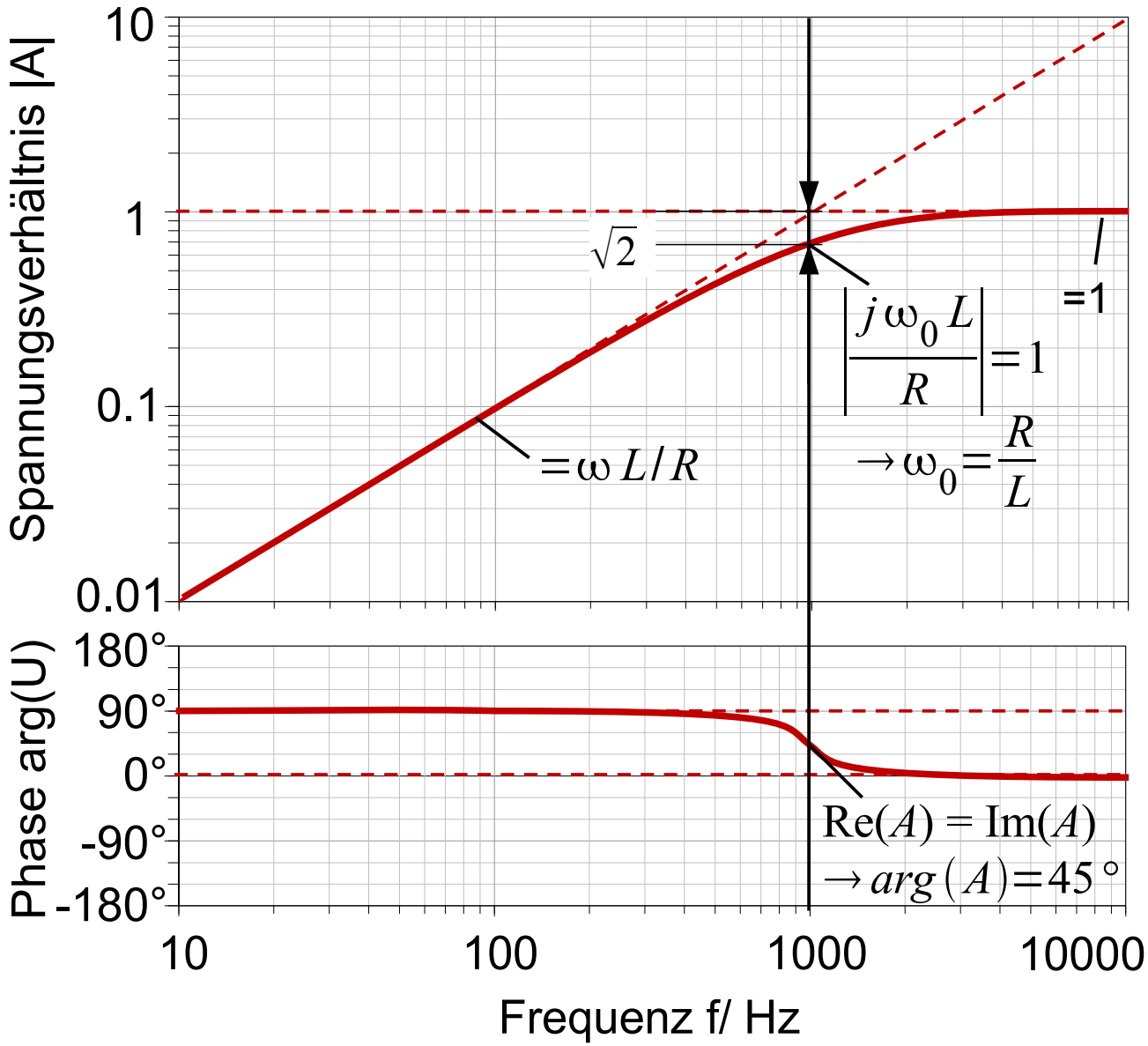


$$\frac{\underline{U}_{aus}}{\underline{U}_{ein}} = \underline{A} = \frac{j\omega \cdot L}{R + j\omega \cdot L}$$

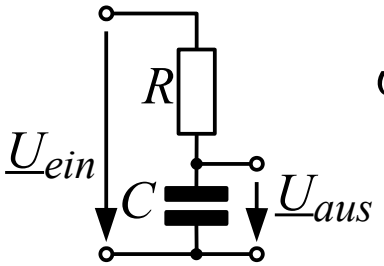
$$= \frac{1}{\frac{R}{j\omega \cdot L} + 1} = \frac{1}{\frac{1}{j\omega L/R} + \frac{1}{1}}$$

$$= j\omega L/R \parallel 1$$

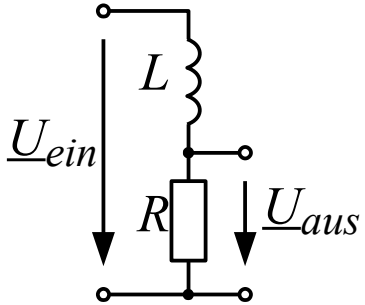
$$|\underline{A}| = \frac{1}{\sqrt{\left(\frac{1}{\omega L/R}\right)^2 + \left(\frac{1}{1}\right)^2}}$$



Hoch- und Tiefpass: Übersicht

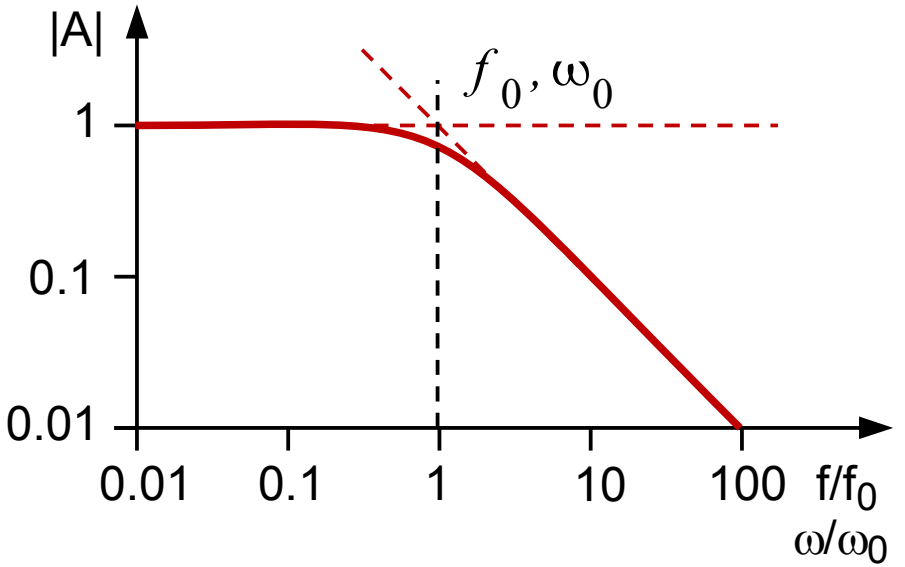


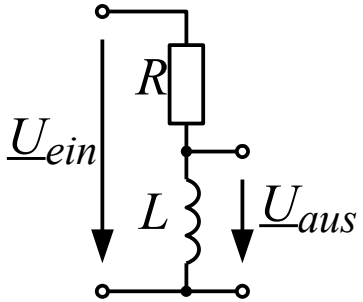
$$\omega_0 = \frac{1}{R \cdot C}$$



$$\omega_0 = \frac{R}{L}$$

$$\underline{A} = \frac{1}{1 + j\,\omega/\omega_0}$$





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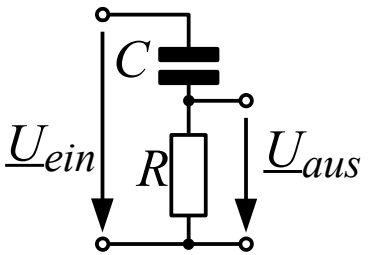
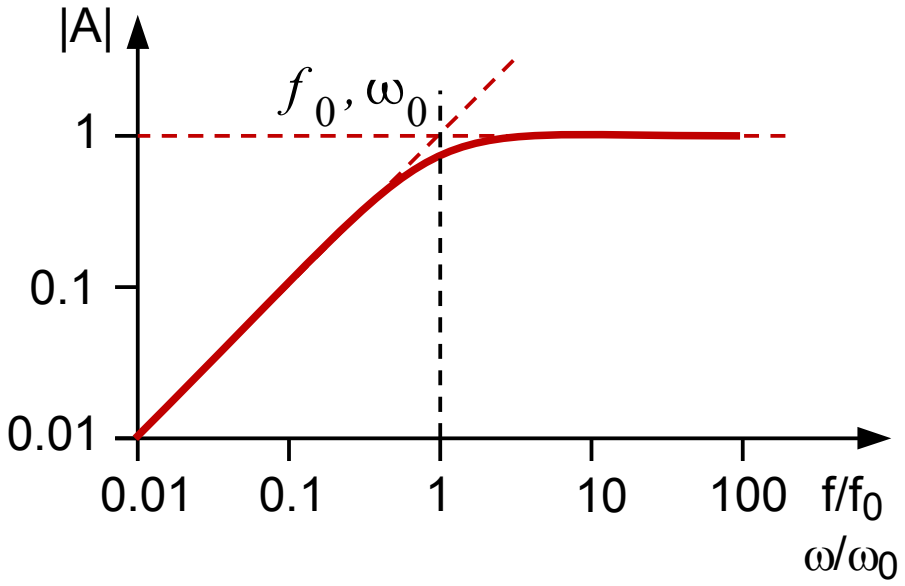


$$\omega_0 = \frac{1}{R \cdot C}$$

$$\underline{A} = \frac{1}{1 + \dfrac{1}{j\,\omega/\omega_0}}$$



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