Grundlagen der Elektrotechnik

Allgemeine komplexe Impedanzen

TH-Köln 2020

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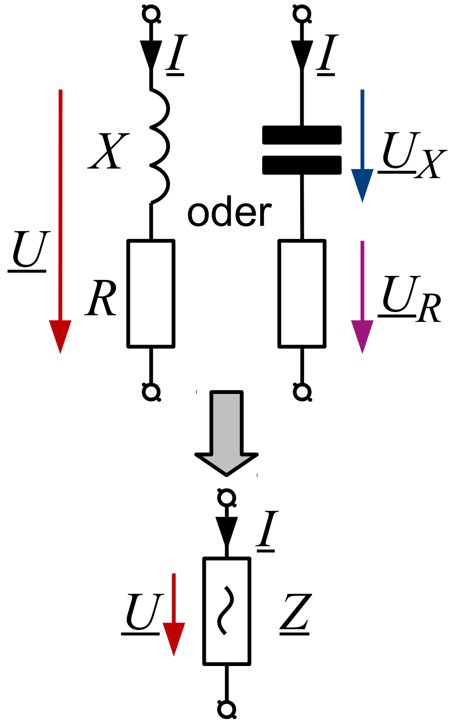
Allgemeine komplexe Impedanzen

- Herleitung allgemeine komplexe Impedanz
- Blindwiderstand und weitere Begriffe
- Rechnen mit Impedanzen
  - Spannungsteiler
  - Parallelschaltung
  - Serien-Parallel-Umwandlung

Allgemeine komplexe Impedanz



$$\underline{U}_X = \underline{I} \cdot jX$$

$$\underline{U}_R = \underline{I} \cdot R$$

Kirchhoff'sche Maschengleichung

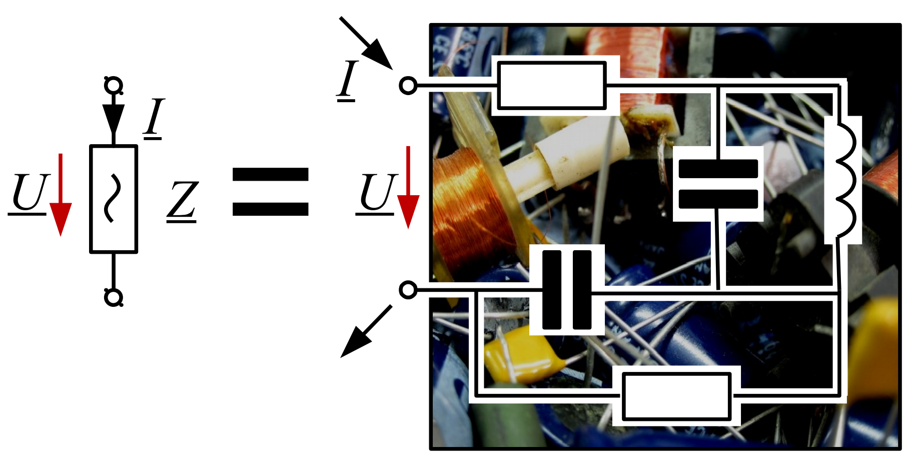
$$\underline{U} = \underline{U}_R + \underline{U}_X$$

$$= \underline{I} \cdot R + \underline{I} \cdot jX$$

$$= \underline{I} \cdot \underbrace{(R + jX)}$$

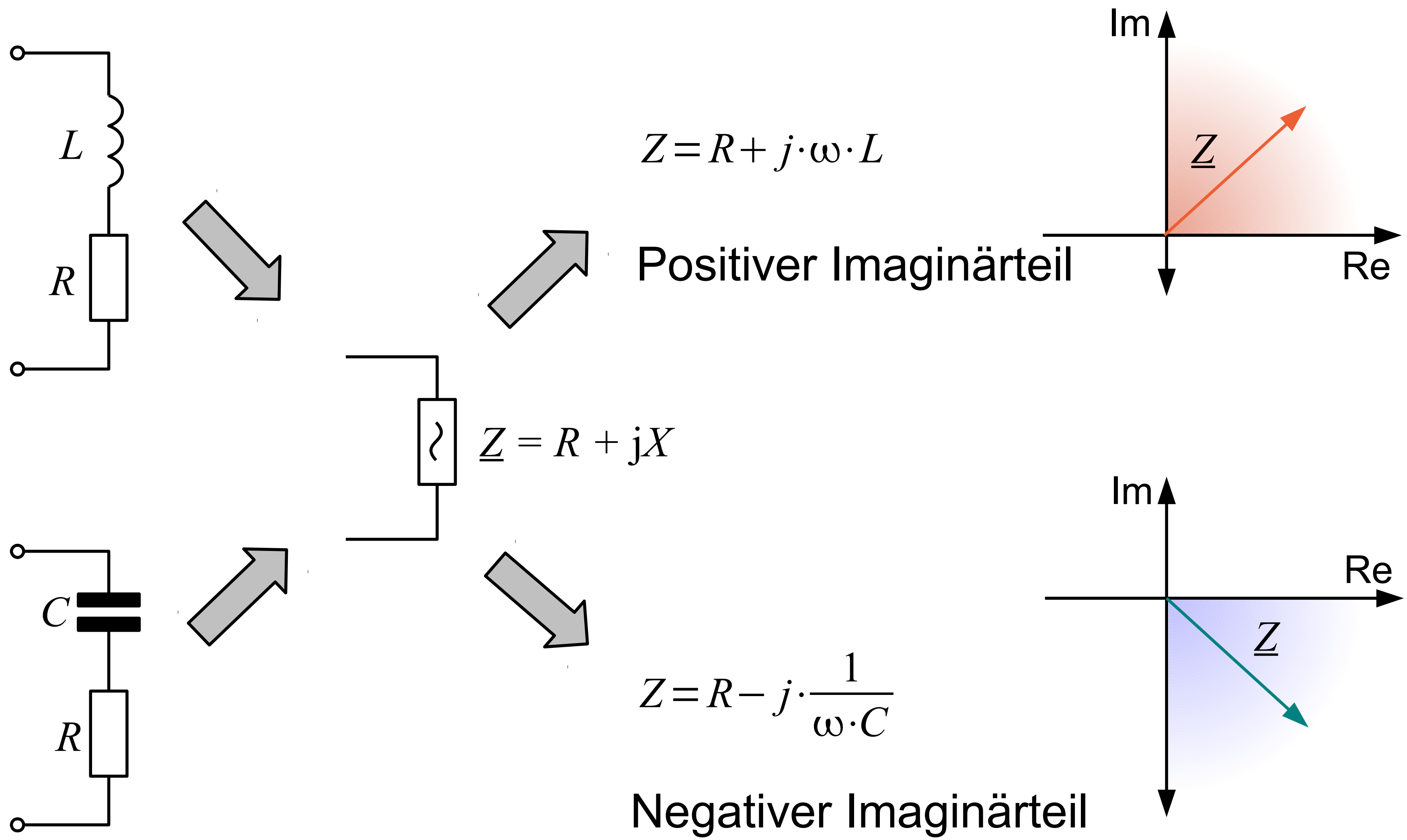
$\boxed{\underline{Z} = R + \mathrm{j}X}$ = komplexe Impedanz

$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}}$ = Allgemeines Ohm'sches Gesetz für komplexe Zeiger

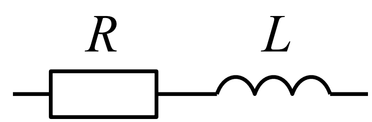
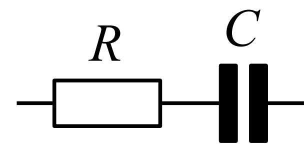


- Komplexe Impedanz beschreibt beliebiges Netzwerk
- Bedingungen:
  - Linear
  - Zeitinvariant
  - Bei einer Frequenz

Allgemeine komplexe Impedanz

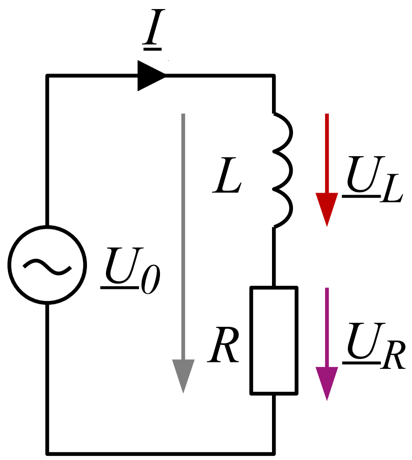


Begriffsdefinitionen

		
Komplexe Impedanz:	$\underline{Z} = R + jX_L$	$\underline{Z} = R - jX_C$
Scheinwiderstand: (Betrag der Impedanz)	$\lvert\underline{Z}\rvert = \sqrt{R^2 + X_L^2} = \frac{\lvert\underline{U}\rvert}{\lvert\underline{I}\rvert}$	$\lvert\underline{Z}\rvert = \sqrt{R^2 + X_C^2} = \frac{\lvert\underline{U}\rvert}{\lvert\underline{I}\rvert}$
Blindwiderstand:	$\lvert\mathrm{Im}(\underline{Z})\rvert = X_L = \omega \cdot L$	$\lvert\mathrm{Im}(\underline{Z})\rvert = X_C = \frac{1}{\omega \cdot C}$
Wirkwiderstand:	$\mathrm{Re}(\underline{Z}) = R$	$\mathrm{Re}(\underline{Z}) = R$

Rechnen mit Impedanzen

Spannungsteiler



$$\underline{U}_L = \underline{I} \cdot \mathrm{j}\omega L$$

$$\underline{U}_R = \underline{I} \cdot R$$

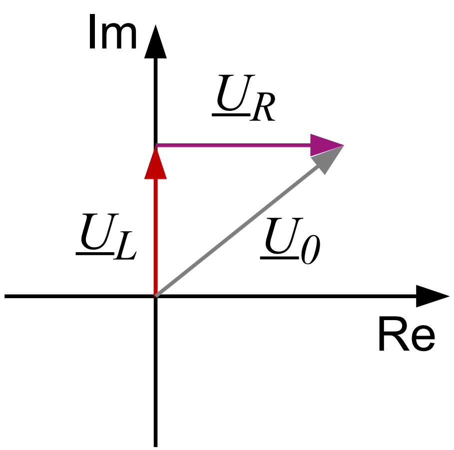
Kirchhoff'sche Maschengleichung

$$\underline{U} = \underline{U}_L + \underline{U}_R$$

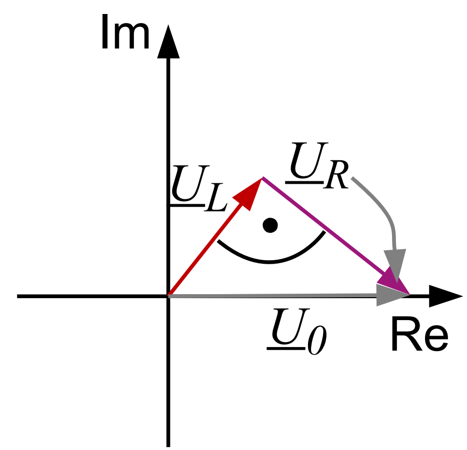
$$\frac{\underline{U}_R}{\underline{U}_0} = \frac{R}{R + jX_L}$$

$$\frac{\underline{U}_L}{\underline{U}_0} = \frac{jX_L}{R + jX_L}$$

mit $X_L = \omega \cdot L$

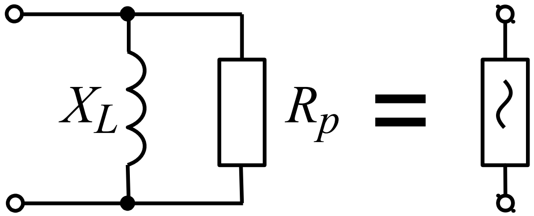


Alternative Darstellung

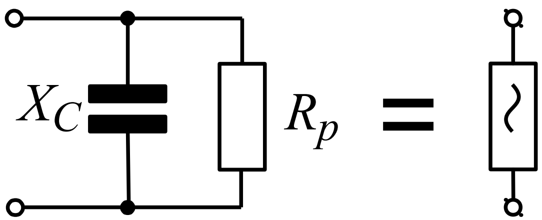


$$|\underline{U}_0|^2 = |\underline{U}_L|^2 + |\underline{U}_R|^2$$

Parallelschaltung

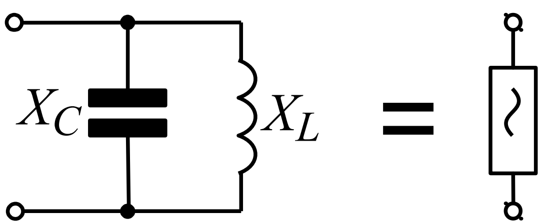


$$\underline{Z} = \frac{1}{\frac{1}{R_P} + \frac{1}{jX_L}} = R_S + jX_S$$



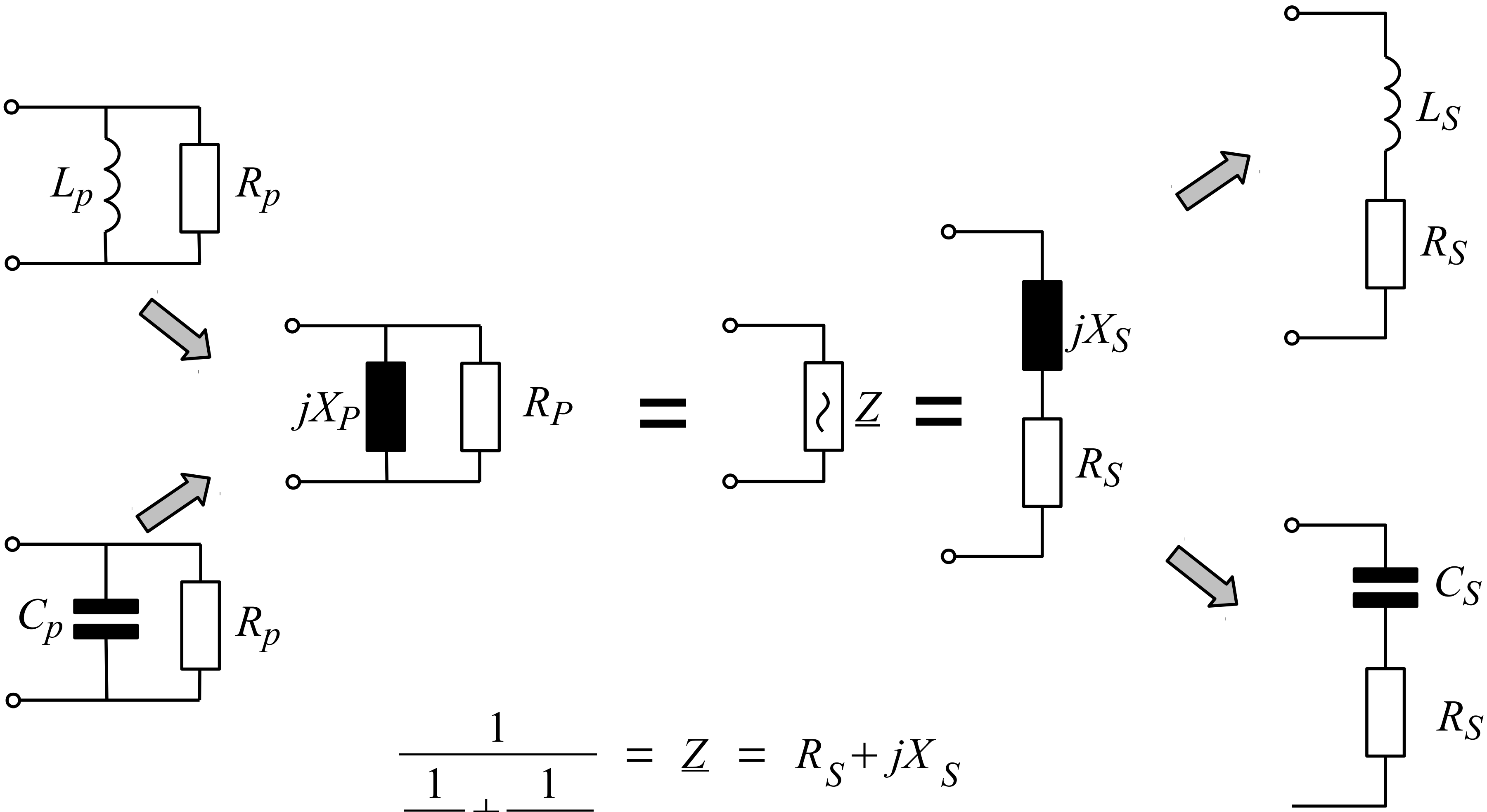
$$\underline{Z} = \frac{1}{\frac{1}{R_P} - \frac{1}{jX_C}} = R_S - jX_S$$

Details siehe Anhang



$$\underline{Z} = \frac{1}{\frac{1}{jX_L} - \frac{1}{jX_C}} = 0 \pm jX_S$$

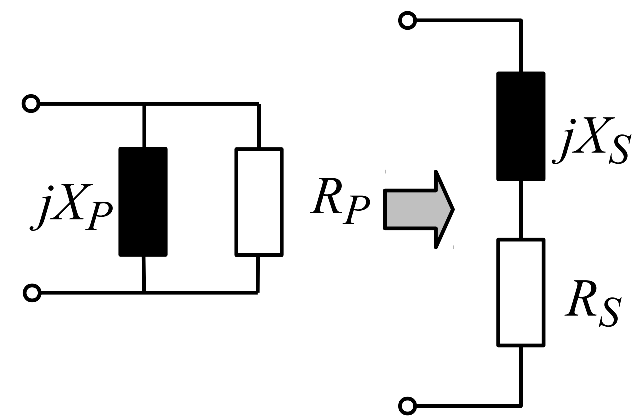
Parallel – Serien - Umwandlung

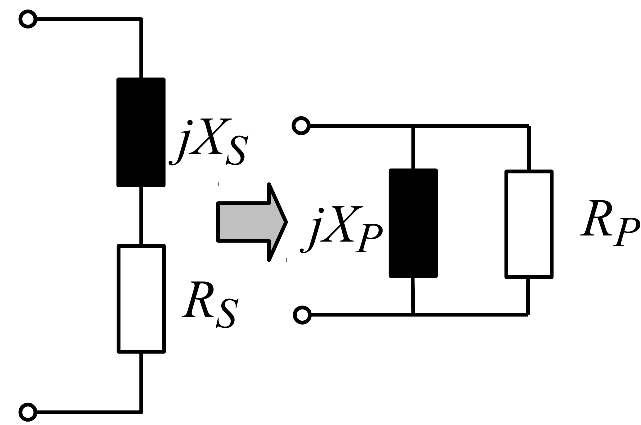


$$\frac{1}{\frac{1}{R_P}+\frac{1}{jX_P}} = \underline{Z} = R_S + jX_S$$

Parallel – Serien - Umwandlung

Bedingung	Formeln und Vereinfachungen	
allgemein	$R_S=\frac{1/R_P}{(1/R_P)^2+(1/X_P)^2}$	$X_S=\frac{1/X_P}{(1/R_P)^2+(1/X_P)^2}$
$R_P \gg X_P$	$R_S=\frac{X_P^2}{R_P}$	$X_S=X_P$
$R_P \ll X_P$	$R_S=R_P$	$X_S=\frac{R_P^2}{X_P}$
allgemein	$\frac{1}{R_p}=\frac{R_S}{R_S^2+X_S^2}$	$\frac{1}{X_p}=-\frac{X_S}{R_S^2+X_S^2}$
$R_S \gg X_S$	$R_P=R_S$	$X_P=\frac{R_S^2}{X_S}$
$R_S \ll X_S$	$R_P=\frac{X_S^2}{R_S}$	$X_P=X_S$





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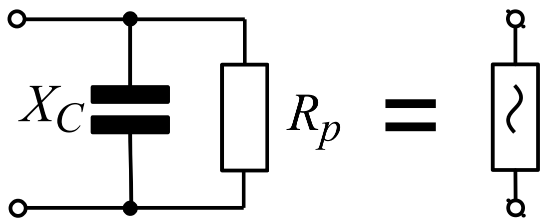
Technology
Arts Sciences
TH Köln

Anhang

Parallelschaltung mit Kondensator



$$\underline{Z} = \frac{1}{\frac{1}{R_P} - \frac{1}{jX_C}} = R_S - jX_C$$

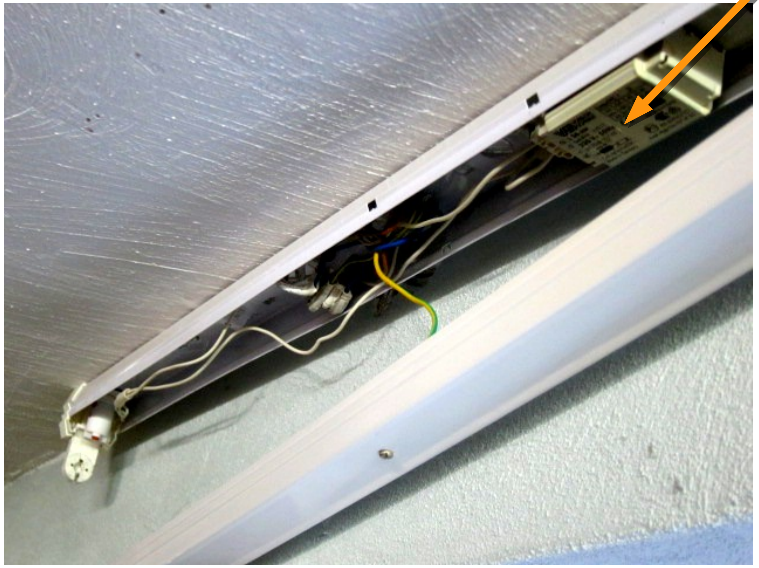
Konjugiert komplex erweitern

$$\underline{Z} = \frac{1}{\frac{1}{R_P} + \frac{1}{-jX_C}} = \frac{1}{\frac{1}{R_P} + j\frac{1}{X_C}} = \frac{\frac{1}{R_P} - j\frac{1}{X_C}}{\left(\frac{1}{R_P} + j\frac{1}{X_C}\right)\cdot\left(\frac{1}{R_P} - j\frac{1}{X_C}\right)} =$$

Binomische Formel

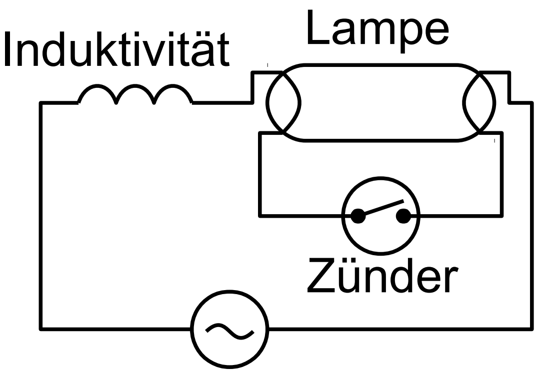
$$\frac{\frac{1}{R_P} - j\frac{1}{X_C}}{\left(\frac{1}{R_P}\right)^2 + \left(\frac{1}{X_C}\right)^2} = \frac{\frac{1}{R_P}}{\left(\frac{1}{R_P}\right)^2 + \left(\frac{1}{X_C}\right)^2} - j\cdot\frac{\frac{1}{X_C}}{\left(\frac{1}{R_P}\right)^2 + \left(\frac{1}{X_C}\right)^2} = R_S - jX_C$$

Anwendung: Leuchtstofflampe

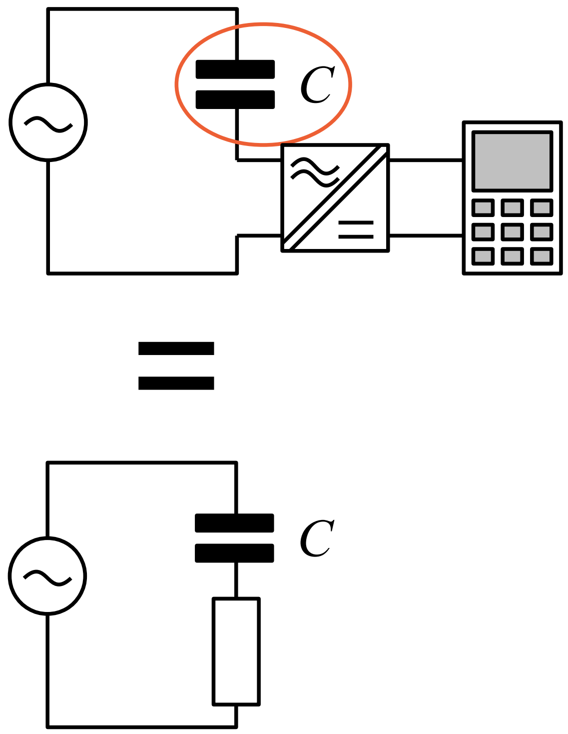


Induktivität

- Begrenzt Strom
- Hilft beim Zünden

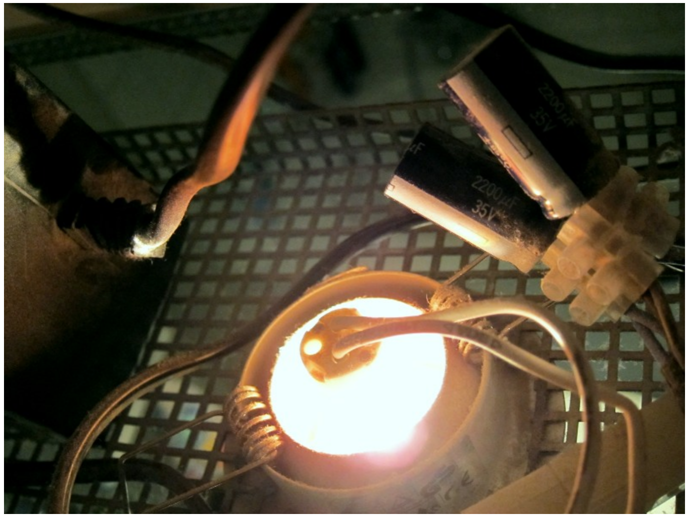


Anwendung: Handy-Lader

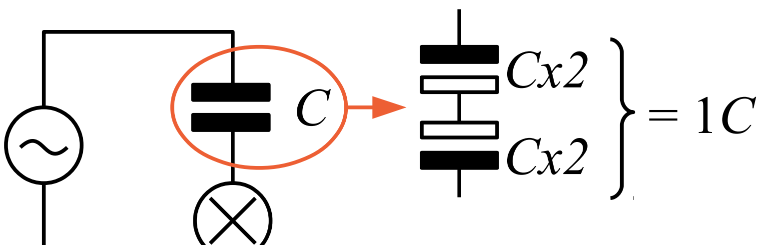


- Kondensator im Spannungsteiler
- Verlustarm
- Geringere Spannung im Netzteil:
  kleiner, preiswerter

Anwendung: Halogenlampe



- Nur 5% Überspannung halbiert die Lebensdauer!



anti-seriell